1) Complete the following life table.

<table>
<thead>
<tr>
<th>Age</th>
<th>( m_x )</th>
<th>( P_x )</th>
<th>( l_x )</th>
<th>( E_x )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0.5</td>
<td>1</td>
<td>( (1.0+0.5+0.1)/1 = 1.6 )</td>
</tr>
<tr>
<td>1</td>
<td>1.8</td>
<td>0.2</td>
<td>0.5</td>
<td>( (0.5+0.1)/0.5 = 1.2 )</td>
</tr>
<tr>
<td>2</td>
<td>0.4</td>
<td>0</td>
<td>0.1</td>
<td>( 0.1/0.1 = 1.0 )</td>
</tr>
</tbody>
</table>

A) Calculate \( R_0 \) and the generation time.

\[
R_0 = \sum l_x m_x = 0.94
\]

\[
T = \frac{\sum x l_x m_x}{R_0} = \frac{0.98}{0.94} = 1.04
\]

B) Construct the Leslie matrix that corresponds to this life table.

\[
\begin{pmatrix}
0.5 \times 1.8 & 0.2 \times 0.4 & 0 \\
0.5 & 0 & 0 \\
0 & 0.2 & 0
\end{pmatrix}
= \begin{pmatrix}
0.9 & 0.08 & 0 \\
0.5 & 0 & 0 \\
0 & 0.2 & 0
\end{pmatrix}
\]

C) Given the initial population densities of \( N_0 = 50 \), \( N_1 = 100 \), and \( N_2 = 25 \), determine \( N \) for the next three generations.

<table>
<thead>
<tr>
<th>( t )</th>
<th>( N_0 )</th>
<th>( N_1 )</th>
<th>( N_2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>50</td>
<td>100</td>
<td>25</td>
</tr>
<tr>
<td>1</td>
<td>53</td>
<td>25</td>
<td>20</td>
</tr>
<tr>
<td>2</td>
<td>49.7</td>
<td>26.5</td>
<td>5</td>
</tr>
<tr>
<td>3</td>
<td>46.85</td>
<td>24.85</td>
<td>5.3</td>
</tr>
</tbody>
</table>

D) Calculate the finite rate of increase and the intrinsic rate of increase for this population.

Since \( R_0 = 1.0 \), the approximation formula may be used:

\[
r = \frac{\ln R_0}{T} = -0.06
\]

\[
\lambda = e^r = 0.9424
\]

E) Determine the stable age distribution for this population.

\[
c_j = \frac{l_j e^{-\eta}}{\sum l_x e^{-\eta}}
\]

\[
c_0 = \frac{1.0 e^{(0.06+0.0)}}{1.0 e^{(0.06+0.0)} + 0.5 e^{(0.06+0.0)} + 0.1 e^{(0.06+2.0)}} = 0.608
\]

\[
c_1 = 0.323
\]

\[
c_2 = 0.0685
\]

F) By how much would the probability of surviving the first year have to be increased for the population to persist indefinitely?
For the population to persist indefinitely \( R_0 \) must equal 1, therefore

\[ l_0 m_0 + l_1 m_1 + l_2 m_2 = 1.0 \]

\[
0.0 + (l_1 \times 1.8) + (0.1 \times 0.4) = 1.0 \\
(l_1 \times 1.8) + 0.04 = 1.0 \\
l_1 \times 1.8 = 0.96 \\
l_1 = \frac{0.96}{1.8} = 0.5333
\]

So, \( l_1 \) must be increased by \( 0.0333 \) to have a population that can persist indefinitely (note that \( P_0 = l_1 \)).

G) Suppose that \( P_0 \) takes on this new value, calculate the reproductive values for each age class.

(note that \( v_0 = R_0 \))

\[
v_0 = m_0 + \frac{l_1}{l_0} m_1 + \frac{l_2}{l_0} m_2 = 1.0 \\
v_1 = m_1 + \frac{l_2}{l_1} m_2 = 1.8 + \frac{0.1}{0.533} 0.4 = 1.875 \\
v_2 = m_2 = 0.4
\]

H) What is the residual reproductive value for an individual starting his second year of life?

\[
v_{1^*} = \frac{l_2}{l_1} v_2 = \frac{0.1}{0.533} 0.4 = 0.075
\]

I) Is it possible to estimate the total population size twenty generations in the future? If so, explain how and state any assumptions that you have made.

Assuming constant \( l_x \) and \( m_x \) schedules, yes, if given the initial population vector. If \( l_1 = 0.5333 \) then \( R_0 = 1.0 \) and the population will not change in size (the births and deaths will cancel each other out exactly). Thus, the population size is the same now as any time in the future.

2) Let \( N = (15, 20, 25, 9) \) and assume that the population has acquired a stable age distribution.

A) If the finite rate of increase is equal to 1.2 per year, estimate the fraction (#) of the total population that will be in the third age class \( (N_3, \text{which currently} = 9) \) in 42 years.

\[
N_{3(t=42)} = \lambda^{42} N_{3(t=0)} = 1.2^{42} \times 9 = 19048.24
\]

B) What is the total size of the population in 18 years?

\[
\text{total population size}\ (t = 0) = \sum_{x=1}^{n} N_x = 69 \\
in 18 \text{ years}, N_{\text{total}(t=18)} = \lambda^{18} N_{\text{total}(t=0)} = 1.2^{18} \times 69 = 1837.01
\]

C) Is it possible to predict the size of the second age class \( (N_2, \text{which currently} = 25) \) in four generations?
No, you do not know the generation time (T) and you cannot find it. [Consider this: how would your answer change if the generation time were 2 days, 2 months, or 2 years? Without knowing the generation time you cannot relate this question to the information that you were given for \( \lambda \) in years.]

D) Calculate the intrinsic rate of increase for this population.

\[
 r = \ln \lambda = \ln(1.2) = 0.182
\]

3) Using the following Leslie matrix, deduce whether the population is increasing, decreasing, or not changing.

\[
\begin{bmatrix}
0.18 & 0.4 & 1.0 & 0.1 & 0.0 \\
0.9 & 0.00 & 0.00 & 0.00 & 0.00 \\
0.00 & 0.8 & 0.00 & 0.00 & 0.00 \\
0.00 & 0.00 & 0.5 & 0.00 & 0.00 \\
0.00 & 0.00 & 0.00 & 0.2 & 0.00 \\
\end{bmatrix}
\begin{bmatrix}
P_0 m_1 & P_1 m_2 & P_2 m_3 & P_3 m_4 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & P_1 & 0 & 0 & 0 \\
0 & 0 & P_2 & 0 & 0 \\
0 & 0 & 0 & P_3 & 0 \\
\end{bmatrix}
\]

<table>
<thead>
<tr>
<th>x</th>
<th>( m_x )</th>
<th>( l_x )</th>
<th>( m_x l_x )</th>
<th>( P_x )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0.9 (= ( l_1/l_0 = 0.9/1.0 ))</td>
</tr>
<tr>
<td>1</td>
<td>0.2</td>
<td>0.9</td>
<td>0.18</td>
<td>0.8 (= ( l_2/l_1 = 0.72/0.9 ))</td>
</tr>
<tr>
<td>2</td>
<td>0.5</td>
<td>0.72</td>
<td>0.36</td>
<td>0.5 (= ( l_3/l_2 = 0.36/0.72 ))</td>
</tr>
<tr>
<td>3</td>
<td>2</td>
<td>0.36</td>
<td>0.72</td>
<td>0.2 (= ( l_4/l_3 = 0.072/0.36 ))</td>
</tr>
<tr>
<td>4</td>
<td>0.5</td>
<td>0.072</td>
<td>0.036</td>
<td>0</td>
</tr>
</tbody>
</table>

\[
R_0 = \xi m_x l_x = 1.296
\]

Since \( R_0 > 1 \), the population size is increasing.